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FRACTURE CRITERION AND DURABILITY OF BRITTLE MATERIALS UNDER CONDITIONS OF STATIONARY HEAT AND MASS TRANSFER

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The equation determining the stochastic character of the development of brittle fracture of a loaded material as a result of the extension of a fracture crack in it in the presence of stationary heat and mass transfer has been obtained. It is shown that the evolution of the fracture crack is determined by the increment of the thermodynamic potential of the material with a change in its characteristic dimension (length). Based on the indicated equation, an estimate of the average durability of a loaded material with an internal rectilinear crack under the conditions of stationary heat and mass transfer is made. From the condition of the stationarity of the potential the brittle fracture criterion of the material and the possibility of closing the fracture crack have been established.

Keywords: *brittleness, destruction, crack, stochasticity, heat and mass transfer, stationarity, durability.*

Introduction. In [1] a thermodynamic approach to the formulation of the criterion of brittle fracture of materials under a stationary thermomechanical action was developed; in [2] it was developed for the case of exposure of a material in a brittle state to the joint (complex) action of stationary mechanical, temperature, and concentration (swelling during moisture absorption) fields.

The most dangerous type of destruction of materials is brittle fracture [1], since it begins instantaneously in the absence of any external evidence that would point to the impending destruction. The majority of articles are maintained in that field of complex external action on them, where from the viewpoint of the mechanics of a deformed solid body no destruction occurs whatever the duration of the external influence (the region of weak external effect).

In the overwhelming majority of experimental and theoretical works on the strength and durability of materials [3, 4], destruction is investigated in the region of tensile stresses σ where there is a linear dependence of the durability logarithm τ on stress σ. In the indicated region of the values of σ the average durability is short (usually not longer than 10^4 sec) and evidently does not correspond to the needed reliability requirements. Thus, the region of small external loadings has been rather inadequately investigated. Therefore of primary interest is the study of brittle fracture in the region of a weak external action in which, as is shown in [3], from the kinetic point of view destruction is possible also by virtue of the thermofluctuation character of the rupture of bonds. In [3, 5] it was established that on exposure of a brittle material to mechanical action the state of dynamic equilibrium and its equivalent condition of the stationarity of the thermodynamic potential $\Delta\Phi$ are shared, from the viewpoint of the material durability, between the regions of active (where $\Delta\Phi$ decreases) and passive (where $\Delta\Phi$ increases) destruction. The question arises as to whether the condition of the stationarity of $\Delta\Phi$ will determine the regions of active and passive development of the brittle fracture process under the joint effect of stationary mechanical, temperature, and concentration fields.

Continuing my work in [1, 2], in the present work, on the basis of the thermodynamics of nonequilibrium processes, an equation for the evolution of the fracture crack was obtained which, as it turned out, is determined by the derivative of the thermodynamic potential with respect to the parameter that characterizes this evolution. Then, the problems of stationary heat and mass transfer are considered for a plane with a cut of zero thickness and length l (rectilinear internal crack) on the edges of which the assigned values of the temperature and concentration of a diffusing substance are maintained, as well as the problems of elasticity theory with account for a constant tensile stress σ directed normally to the cut. Then, an expression for the thermodynamic potential of a sample $\Delta\Phi$ is obtained, from the

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stationarity condition of which the corresponding material brittle fracture criterion and the conditions of its retardation by a temperature and (or) concentration fields were obtained.

Statement of the Problem. We will consider an element of volume dV of the material. For the sake of simplicity we will characterize the evolution of a crack by a single parameter l. Generalization to the case of more than one evolution parameter does not present significant difficulties.

In the process of evolution of the system placed into a thermostat (in the case considered this is a crack-containing material located in the surrounding medium) to a new equilibrium state in time dt the following processes proceed: heat and mass exchange of the system with the thermostat (medium), transition of the system from the state in which the fracture crack has the value of the evolution parameter equal to l to the state with the value of the evolution parameter $l + dl$. After the indicated transition, in the material due to the rupture of bonds a new surface appears through which additional heat- and mass exchange of the system with an external heat and mass source (determined by the corresponding boundary conditions) occurs and the external mechanical field (e.g., tensile stress) performs a definite work which, just as the heat- and mass exchange, alters the strained-stressed state of the material. According to this and on the assumption of local quasiequilibrium the change of the enthalpy $w = \varphi + Ts$ of the unit volume of the material in time dt can be presented in the form $d_t w = d_t\varphi + s d_t T + T d_t s$. Since the change of $d_t\varphi$ occurs as a result of the change in the temperature field, in the number of the particles that penetrated into the material, and of the change in the crack length l, $d_t\varphi = d_T\varphi + d_n\varphi + d_l\varphi$, then

$$d_t w = d_T\varphi + d_n\varphi + d_l\varphi + s d_t T + T d_t s\,. \tag{1}$$

The subscripts at the differentials signify, as is customary, partial differentials with respect to the corresponding variables. Since $d_T\varphi = -s d_t T$, equality (1) can now be presented in the form

$$d_t w = d_n\varphi + d_l\varphi + T d_t s\,. \tag{2}$$

Expressing the partial differentials in terms of the partial derivatives and differentials of independent arguments and after division by dt, we obtain

$$\frac{\partial w}{\partial t} = \frac{\mu}{v_{\mathrm{a}}}\frac{\partial n}{\partial t} + \frac{\partial\varphi}{\partial l}\frac{dl}{dt} + T\frac{\partial s}{\partial t}\,. \tag{3}$$

Under the conditions of a constant external loading applied to the system surface $\frac{\partial w}{\partial t} = \operatorname{div}(\lambda\nabla T)$. Moreover, $\frac{\partial n}{\partial t} = \operatorname{div}(D\nabla n)$. Multiplying both sides of Eq. (3) by dV and integrating over the entire volume of the system V, we obtain the following equation for the rate of change in the system's entropy $S = \iiint\limits_V s dV$:

$$\frac{dS}{dt} = \iiint\limits_V \frac{1}{T}\operatorname{div}(\lambda\nabla T)\,dV - \iiint\limits_V \frac{\mu}{v_{\mathrm{a}}T}\operatorname{div}(D\nabla n)\,dV - \iiint\limits_V \frac{1}{T}\frac{\partial\varphi_0}{\partial l}\,dV\frac{dl}{dt} - \iiint\limits_V \frac{1}{T}\frac{\partial\varphi_1}{\partial l}\,dV\frac{dl}{dt}\,. \tag{4}$$

The first three terms on the right-hand side of Eq. (4) represent the rate of change in the entropy of the system as a result of heat- and mass transfer in the volume of the system's material attributable to the action of a corresponding external source. Their contribution to the value of $\frac{dS}{dt}$ can be both positive and negative and is not directly connected with the entropy production due to the system relaxation to the state of equilibrium with a change in the evolution parameter l. Unlike the first three terms, the fourth term in Eq. (4) represents just the power of the entropy relaxation source caused by the transition of the system to a more equilibrium state as a result of the change in the evolution parameter l. According to the second law of thermodynamics, the relaxation source power must be nonnegative, i.e.,

$$-\iiint\limits_V \frac{1}{T}\frac{\partial \varphi_1}{\partial l}\, dV \frac{dl}{dt} \ge 0\,. \tag{5}$$

The simplest condition that ensures the fulfillment of inequality (5) is the relation

$$\frac{dl}{dt} = -\frac{\gamma\lambda_0}{k_{\mathrm{B}}T_{\mathrm{m}}}\frac{d\iiint\limits_V \varphi_1 dV}{dl}\,. \tag{6}$$

Note that the evolution equation in the form $\frac{dl}{dt} = -\frac{\gamma\lambda_0}{k_{\mathrm{B}}}\iiint\limits_V \frac{1}{T}\frac{\partial\varphi_1}{\partial l}\, dV$ is more complex, but it also does not contradict condition (5) and virtually coincides with the condition (6) if $\frac{|T - T_0|}{T_0} << 1$. Usually it is realized in practice and corresponds to the condition of the applicability of the thermoelasticity conditions. Since $\iiint\limits_V \varphi_1 dV = \Phi_1$, the equation of the system evolution due to the change in the evolution parameter l will take the form

$$\frac{dl}{dt} = -\frac{\gamma\lambda_0}{k_{\mathrm{B}}T_{\mathrm{m}}}\frac{d\Phi_1}{dl}\,. \tag{7}$$

The evolution equation (7) makes it possible not only to establish the criterion for the beginning of the irreversible evolution of the system, but also to find the average time of transition of the system to the active phase of evolution from the state in which $\frac{dl}{dt} < 0$, since it allows one to compose the kinetic equation of the evolution of the system that accounts for its stochastic character. Indeed, the quantity $\frac{dl}{dt}$ can also be presented in the form

$$\frac{dl}{dt} = \lambda_0\,(w^+ - w^-)\,. \tag{8}$$

Having defined the diffusion coefficient D in the space of the values of the evolution parameter l by the expression

$$D = \frac{1}{2}\lambda_0^2\,(w^+ - w^-)\,, \tag{9}$$

and the coefficient γ by the formula $\gamma = w^+$, we obtain the following expression for the diffusion coefficient D:

$$D = w^+\lambda_0^2\left(1 + \frac{\lambda_0}{k_{\mathrm{B}}T_{\mathrm{m}}}\frac{d\Phi_1}{dl}\right). \tag{10}$$

Then the equation for the probability $P(l, t)$ that the system at time t will occur in the state between the values of the evolution parameter l and $l + dl$ will have the form

$$\frac{\partial P\,(l, t)}{\partial t} = \frac{\partial}{\partial l}\left(P\,(l, t)\frac{w^+\lambda_0}{k_{\mathrm{B}}T_{\mathrm{m}}}\frac{d\Phi_1}{dl}\right) + \frac{\partial^2}{\partial l^2}\left(P\,(l, t)\, w^+\lambda_0^2\left(1 + \frac{\lambda_0}{2k_{\mathrm{B}}T_{\mathrm{m}}}\frac{d\Phi_1}{dl}\right)\right).$$

The thermodynamic potential of the system Φ_1 entering into Eq. (7) was calculated proceeding from the strained-stressed state of the system originating due to the action of corresponding fields on the system [1].

Calculations. According to [1, 2], the thermodynamic potential of the system Φ_1 is defined by the relation

$$\Phi_1 = -\frac{1}{2}\iiint\limits_V \sigma_{ik}\varepsilon_{ik}dV - \frac{1}{2\,(1-2\nu)}\,E\left(\beta_C\iiint\limits_V (C-C_0)\,\varepsilon_{ll}dV + \alpha_T\iiint\limits_V (T-T_0)\,\varepsilon_{ll}dV\right) + \iint\limits_{S_{\text{cr}}} \alpha_{\text{s}}dS\,. \tag{11}$$

Here, the temperature and concentration fields are the solution of the following boundary-value problem of stationary heat and mass transfer:

$$\Delta W(\text{M}) = 0\,,\quad \text{M} \in E^3 \setminus S_{\text{cr}}\,, \tag{12}$$

$$W(\text{M}) = W_1\,,\quad \text{M} \in S_{\text{cr}}\,, \tag{13}$$

$$\lim_{\rho(\text{M})\to\infty} W(\text{M}) = 0\,. \tag{14}$$

To determine the fracture criterion of brittle materials under the conditions of stationary heat and mass transfer it is sufficient to know the increment of the thermodynamic potential of the material $\Delta\Phi_1$ relative to the state of this very material, but without a crack. We will substract, from Φ_1, the indicated value of the thermodynamic potential $\Phi_1^{(1)} = \frac{1}{2}\iint\limits_V \sigma_{ik}^{(1)}\varepsilon_{ik}^{(1)}dV$, where $\sigma_{ik}^{(1)} = \sigma$ and $\varepsilon_{ik}^{(1)}$ is the deformation tensor corresponding to the stressed state that appears in the system without a crack under the conditions of the action of σ. As a result, we arrive at the following expression for $\Delta\Phi_1$:

$$\Delta\Phi_1 = -\iiint\limits_V \sigma_{ik}^{(2)}\varepsilon_{ik}^{(1)}dV - \frac{1}{2}\iiint\limits_V \sigma_{ik}^{(2)}\varepsilon_{ik}^{(2)}dV + \iint\limits_{S_T} \alpha_{\text{s}}dS - \frac{E}{2\,(1-2\nu)}\iiint\limits_V W_0\,(\text{div}\,\mathbf{U}^{(2)} + 2\,\text{div}\,\mathbf{U}^{(1)})\,dV\,. \tag{15}$$

In the derivation of Eq. (15) it was taken into account that $\sigma_{ik}^{(1)}\varepsilon_{ik}^{(2)} = \sigma_{ik}^{(2)}\varepsilon_{ik}^{(1)} + \frac{E}{1-2\nu}W_0\ \text{div}\ \mathbf{U}^{(1)}$.

Next, according to [1] we transform the first two integrals over the material volume V into the integrals over the surfaces that bound this volume. With account for the equilibrium condition we have

$$\iiint\limits_V \sigma_{ik}^{(2)}\varepsilon_{ik}^{(2)}dV = \frac{1}{2}\iint\limits_{S_0} \sigma_{ik}^{(2)}n_k u_i^{(2)}dS + \frac{1}{2}\iint\limits_{S_{\text{cr}}} \sigma_{ik}^{(2)}n_k u_i^{(2)}dS = \frac{1}{2}\iint\limits_{S_{\text{cr}}} \sigma_{ik}^{(2)}n_k u_i^{(2)}dS = -\frac{1}{2}\iint\limits_{S_{\text{cr}}} \sigma_{ik}^{(1)}n_k u_i^{(2)}dS\,,$$

since on the outer side of the surface S_0 that bounds the material the condition $\sigma_{ik}^{(1)}n_k = \sigma$ is satisfied and, consequently, $\sigma_{ik}^{(2)}n_k = 0$. On the crack surface S_{cr} in the absence of the forces acting on its edges the condition $\sigma_{ik}^{(2)}n_k = -\sigma_{ik}^{(1)}n_k$, $i = 1, 2, 3$ is satisfied. Then analogously

$$\iiint\limits_V \sigma_{ik}^{(2)}\varepsilon_{ik}^{(1)}dV = \frac{1}{2}\iint\limits_{S_0} \sigma_{ik}^{(2)}n_k u_i^{(1)}dS + \frac{1}{2}\iint\limits_{S_{\text{cr}}} \sigma_{ik}^{(2)}n_k u_i^{(2)}dS = 0\,,$$

since $u_i^{(1)} = 0$ at the supposed cite of appearance of a rectilinear internal crack. Thus, finally we arrive at the following expression for the increment of the thermodynamic potential $\Delta\Phi_1$:

$$\Delta\Phi_1 = \frac{1}{2}\iint\limits_{S_{\text{cr}}} \sigma_{ik}^{(1)}n_k u_i^{(2)}dS - \frac{E}{2\,(1-2\nu)}\iiint\limits_V W_0\,(\text{div}\,\mathbf{U}^{(2)} + 2\,\text{div}\,\mathbf{U}^{(1)})\,dV + \iint\limits_{S_{\text{cr}}} \alpha_{\text{s}}dS\,. \tag{16}$$

It follows from (16) that to calculate the value of $\Delta\Phi_1$ it is necessary to know the components of the displacement vector $\mathbf{U}^{(2)}$, the temperature, W_T, and concentration, W_C, fields and div $\mathbf{U}^{(2)}$, div $\mathbf{U}^{(1)}$. In the present work the value of $\Delta\Phi_1$ is calculated in the case of the plane-stressed state of a thin plate with a rectilinear internal crack of length l modeled by a plane with a cut of zero thickness and length l on the edges of which the assigned values of temperature, $W_{T,1} = T_1 - T_0$, and concentration, $W_{C,1} = C_1 - C_0$, of the substance diffusing from the environment into the material are maintained. Under these conditions both functions W_T and W_C are the solutions of the same boundary-value problem for the Laplace equation which, with account for the symmetricity of the unknown functions over the variables x and y in the system of rectangular Cartesian coordinates with the Ox axis located along the cut and the Oy axis located perpendicularly to the cut in the middle of it, is reduced to a boundary-value problem for the Laplace equation in a semiplane at the boundary of which $x = 0$ different-kind boundary conditions are assigned: $W(x, 0) = W_1$ at $|x| < \frac{l}{2}$ and $\partial W/\partial y = 0$ at $y = 0$, $|x| > \frac{l}{2}$. As a result, for the solution of the problem, we must solve dual integral equations of the type

$$\int_0^\infty \cos(\omega x) f(\omega)\, d\omega = W_1\,, \quad 0 < x < \frac{l}{2}\,; \quad \int_0^\infty \cos(\omega x) f(\omega)\, \omega d\omega = 0\,, \quad x > \frac{l}{2}\,, \tag{17}$$

where $f(\omega)$ is the function that determines the sought solution of the above-indicated boundary-value problems for the Laplace equation from the equation

$$W(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-i\omega x - |\omega|\,|y|\right) f(\omega)\, d\omega\,. \tag{18}$$

The function $f(\omega)$ can be found with the aid of the table of solutions of dual integral equations given in [3]. To find the functions div $\mathbf{U}^{(2)}$ and $u_y^{(2)}$ (since due to the choice of the coordinate system as indicated above $\sigma_{ik}^{(2)} n_k u_i^{(2)} = -\sigma u_y^{(2)}$ and div $\mathbf{U}^{(2)} = \frac{1}{E}(1 - 2\nu)(\sigma_{xx}^{(2)} + \sigma_{yy}^{(2)}) + 3W_0$ in the plane-stressed state) the Fourier transformation over the variable x of the equilibrium equation in stresses was used. As a result, for the quantities $\sigma_{xx}^{(2)}$ and $\sigma_{yy}^{(2)}$ the following expressions were obtained [1]:

$$\sigma_{yy}^{(2)}(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-i\omega x - |\omega|\,|y|\right) A(\omega) \left(1 + |\omega|\,|y|\right) d\omega\,, \tag{19}$$

$$\sigma_{xx}^{(2)}(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-i\omega x - |\omega|\,|y|\right) A(\omega) \left(1 - |\omega|\,|y|\right) d\omega\,. \tag{20}$$

The unknown function $A(\omega)$ is found from the conditions

$$\sigma_{yy}^{(2)} = -\sigma\,, \quad |x| < \frac{l}{2}\,, \tag{21}$$

$$u_y^{(2)}(x, 0) = 0\,, \quad |x| > \frac{l}{2}\,, \tag{22}$$

where

$$u_y^{(2)}(x, y) = -\frac{1}{E\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-i\omega x - |\omega|\,|y|\right) \frac{A(\omega)}{|\omega|} \left(2 \operatorname{sign} y + (1 + \nu)\, |\omega|\, y\right) d\omega$$

$$-\alpha_T \int_y^{\infty} W_T(x, \eta)\, d\eta - \beta_C \int_0^{\infty} W_C(x, \eta)\, d\eta \,. \tag{23}$$

The substitution of the needed expressions into conditions (21) and (22) leads to the following dual integral equation:

$$\int_0^{\infty} g(\omega) \cos(\omega x)\, d\omega = -\sigma\sqrt{2\pi} + \frac{1}{2} E \sqrt{2\pi}\, W_0(x, 0)\,, \quad 0 < x < \frac{l}{2}\,, \tag{24}$$

$$\int_0^{\infty} g(\omega) \cos(\omega x) \frac{d\omega}{\omega} = 0\,, \quad x > \frac{l}{2}\,. \tag{25}$$

Here $g(\omega) = 2A(\omega) + E\overline{W}_0(\omega, 0)$, where $\overline{W}_0(\omega, 0)$ is the Fourier transform of $W_0(x, 0)$. The dual integral equation (24), (25) was solved also with the aid of the table of solutions of dual integral equations [3]. The substitution of the obtained expressions div $\mathbf{U}^{(2)}(x, y)$, $u_y^{(2)}$, $u_y^{(2)}(x, 0)$, and $W_0(x, y)$ into Eq. (16) led to the expression for the increment of the thermodynamic potential $\Delta\Phi_1$:

$$\Delta\Phi_1 = \lambda_p \left(-\frac{\pi\sigma^2 l^2}{4E} - \frac{3\pi E l^2 W_{TC}^2}{16\,(1-2\nu)} + 2\alpha_s l \right). \tag{26}$$

Discussion of Results. On the basis of Eq. (26), from the condition of the extremum of $\Delta\Phi_1$ we obtain expressions for the critical length of the crack l^* that determines the beginning of its active growth:

$$l^* = \frac{4\alpha_s}{\dfrac{\pi\sigma^2}{E} + \dfrac{3\pi E W_{TC}^2}{4\,(1-2\nu)}}\,, \tag{27}$$

and of the threshold of external stress:

$$\sigma^* = \sqrt{\frac{4\alpha_s E}{\pi l} - \frac{3\,(E W_{TC})^2}{4\,(1-2\nu)}}\,, \tag{28}$$

beginning from which, provided that $\sigma > \sigma^*$, the stage of the active growth of the crack of initial length exists. Both these criteria, just as the problem of a mathematical crack considered in the present work, have meaning only when the condition of the positive displacement of the crack edges is satisfied, i.e., $u_y^{(2)}(x, 0) > 0$. The failure of this condition means the joining of the crack edges and the impossibility of its development in the material, i.e., the strengthening of the material. In the conditions considered by us $u_y^{(2)}(x, 0)$ is defined as

$$u_y^{(2)}(x, 0) = \left(\frac{2\sigma}{E} - \alpha_T W_{T,1} - \beta_C W_{C,1} \right) \sqrt{\left(\frac{l^2}{4}\right) - x^2}\,, \quad |x| \le \frac{l}{2}\,.$$

From this is follows that the region of the parameters that characterize the external action on the material that blocks the development of the crack can be found from the inequality $\frac{2\sigma}{E} - \alpha_T W_{T,1} - \beta_C W_{C,1} \le 0$. It is evident that at the external tensile stress this inequality can be satisfied only when at least one of the quantities entering into the inequality, $\alpha_T W_{T,1}$ or $\beta_C W_{C,1}$, is higher than zero.

According to [3, 5], the average durability in the region of external action, where $\sigma < \sigma^*$, is defined by the expression

$$\tau \approx \frac{1}{\lambda_0 w^+ (l^*)} \exp \left(\frac{\Delta\Phi_1 (l^*)}{k_{\mathrm{B}} T_{\mathrm{m}}} \right) \sqrt{\frac{2\pi k_{\mathrm{B}} T_{\mathrm{m}}}{|\Delta\Phi_1'' (l^*)|}} ,$$

where the quantity l^* is defined by Eq. (27), and the frequency of direct transition (i.e., of the rupture of bond) by the equation $w^+(l) = \nu_0 \exp \left(\frac{U - V_{\mathrm{fl}}\sigma_{\mathrm{cr}}(l, \sigma)}{k_{\mathrm{B}} T_{\mathrm{m}}} \right)$ [1]. This very results is obtained also with the aid of the above-given equation for the probability $P(l, t)$ on the basis of the method of finding the average durability proposed in [3, 5].

CONCLUSIONS

1. The equation that models the stochastic process of brittle fracture of a loaded material has been obtained in which stationary heat and mass transfer occurs as a result of the development of a fracture crack in it.

2. It is shown that to find the average durability of the brittle material with the aid of the indicated equation, of the criteria of brittle fracture and strengthening of the material, it is necessary to determine the thermodynamic potential of the system by solving the corresponding problem of the mathematical theory of a crack that accounts for the stationary temperature and concentration fields in the material.

NOTATION

C_0, certain concentration of diffusant which in the absence of external forces in a material is considered nondeformed, mole/m^3; C_1, concentration of diffusant on the crack edges, mole/m^3; C(M), concentration of diffusant at the point M of the material, mole/m^3; D, diffusion coefficient, m^2/sec; E, Young's modulus, N/m^2; k_{B}, Boltzmann constant, J/K; l, crack length, m; l^*, crack length in the state of dynamic equilibrium, m; n, number of diffusant particles in unit volume of the material, mole/m^3; $P(l, t)$, probability of finding a system at time t in the state between the values of the evolution parameter l and $l + dl$; S_{cr}, crack surface; S_0, external surface of material; s, entropy of unit volume of material, J/(deg·m^3); $\operatorname{sign} y = \begin{cases} -1 , & y < 0 \\ 0 , & y = 0, \\ 1 , & y > 0 \end{cases}$ sign function; T_{m}, temperature of the medium, K; T(M), temperature at the point M of the material, K; T_1, temperature on the crack edges, K; T_0, certain assigned temperature at which the system is considered undeformed in the absence of external forces, K; t, time, sec; U, activation energy of the rupture of bonds at $\sigma = 0$, J; $\mathbf{U}^{(2)}$ and $\mathbf{U}^{(1)}$, vectors of displacement in the presence and absence of a crack in the material, m; $u_i^{(2)}$, ith component of the displacement vector $\mathbf{U}^{(2)}$, m; $u_i^{(1)}$, ith component of the displacement vector $\mathbf{U}^{(1)}$, i = 1, 2, 3 or x, y, z, respectively; V, body volume, m^3; V_{fl}, fluctuation volume, m^3; v_{a}, average volume per material particle, m^3; $W_0 = \alpha_T W_T + \beta_C W_C$, $W_{TC} = \alpha_T W_{T,1} + \beta_C W_{C,1}$, $W_T = T(\mathrm{M}) - T_0$, K; $W_C = C(\mathrm{M}) - C$, mole/m^3; $W(\mathrm{M}) = W_T(\mathrm{M})$ or $W(\mathrm{M}) = W_C(\mathrm{M})$, as well as $W_1 = W_{T,1} = T_1 - T_0$ or $W_1 = W_{C,1} = C_1 - C_0$ depending on what boundary-value problem (of heat conduction or diffusion) is considered in (12)–(14); w, enthalpy of unit volume of the material, J/m^3; w^+, frequencies of transition from system's state with the value of the evolution parameter l into system's state with the evolution parameter $l + dl$, sec^{-1}; w^-, frequency of transition from system's state with the value of the evolution parameter $l + dl$ to the state with the evolution parameter l, sec^{-1}; α_{s}, specific free surface energy of material with allowance for adsorption of diffusing substance on the crack surface, J/m^2; α_T, coefficient of linear thermal expansion of material, deg^{-1}; β_C, coefficient of linear expansion of material on swelling, mole^{-1}; $\Delta\Phi$, thermodynamic potential of a sample in the case of pure mechanical loading, J; $\Delta\Phi_1$, change in the thermodynamic potential of the sample Φ_1 relative to its state without a crack, J; ε_{ik}, tensor of deformations; Φ_1, thermodynamic potential of a sample not connected with entropy production by external heat and mass sources, J; φ, thermodynamic potential of a unit volume of material, $\mathrm{J/m^3}$; φ_0, φ_1, densities of the thermodynamic potential of the system related and not related to the production of entropy by external heat and mass sources, $\mathrm{J/m^3}$; γ, kinetic coefficient ($\gamma > 0$), m/sec; λ, thermal conductivity, W/(deg·m); λ_0, characteristic change in the value of evolution parameter in transition of the system from the given metastable state into the neighboring one, m; λ_{p}, length of the crack front participating in its fluctuational motion, m; μ, chemical potential of the substance diffusing into the material (diffusant) and causing its swelling, J; ν, Poisson coefficient; ν_0, frequency of attempts to overcome the energy barrier on the way to rupture of the bonds, sec^{-1}; $\rho(M)$, distance of point M to the coordinate origin, m; σ_{ik}, stress tensor, $\mathrm{N/m^2}$; σ, tensile external stress, $\mathrm{N/m^2}$; $\sigma_{\mathrm{cr}}(l, \sigma)$, stress at the top of the crack, $\mathrm{N/m^2}$; σ^*, external stress determining the beginning of the active growth of a crack, $\mathrm{N/m^2}$; τ, average durability of material, sec. Subscripts: a, atom; C, concentration; cr, crack; fl, fluctuation; m, medium; p, perimeter; s, surface; T, temperature.

REFERENCES

1. V. V. Shevelev, Brittle-fracture criterion and durability of materials under thermomechanical action, *Inzh.-Fiz. Zh.*, **81**, No. 2, 394–400 (2008).
2. V. V. Shevelev, Longevity of brittle materials under the conditions of stationary heat and mass transfer, in: *Abstracts of Papers and Communications of the 6th Minsk Int. Heat and Mass Transfer Forum*, Vol. 1, Minsk (2008), pp. 298–301.
3. É. M. Kartashov, B. Tsoi, and V. V. Shevelev, *Structure-Statistical Kinetics of the Destruction of Polymers* [in Russian], Khimiya, Moscow (2002).
4. V. R. Regel', A. I. Slutsker, and É. E. Tomashevskii, *Kinetic Nature of the Strength of Solid Bodies* [in Russian], Nauka, Moscow (1974).
5. V. V. Shevelev and É. M. Kartashov, Some statistical aspects of brittle destruction and longevity of polymers. *Materials with Cracks*, *Vysokomol. Soedin.*, *Ser. B*, **39**, No. 2, 371–381 (1977).